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Assignment I MTH 512 , Fall 2018

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QUESTION 1. Convince me why the following are not subspaces (short answer)

- F is a field and $D = \{f(x) \in F[x] \mid f(1) = 0 \text{ or } f'(1) = 0\}$

- $D = \{(a, -a, a^2) \mid a \in R\}$.

- F is a field $D = \{A \in F^{2\times 2} \mid |A| = 0\}$.

- F is a field $D = \{A \in F^{3\times 3} \mid rank(A) \leq 2\}$.

QUESTION 2. (short proof) Let V, W be vector spaces over a field such that $IN(V) = IN(W) = n < \infty$ and $T : V \to V$ be a linear transformation . If $Z(T) = \{0_V\}$,(i.e. $IN(Z(T)) = 0$), then Convince me that T is onto and 1-1 (i.e., T is an isomorphism).

(short proof) Let V, W be vector spaces over a field such that $IN(V) = IN(W) = n < \infty$ and $T : V \to V$ be a linear transformation. If T is onto, convince me that T is 1-1.

QUESTION 3. Let $T : R^4 \to R^3$ such that $T(a_1, a_2, a_3, a_4) = (-a_1, a_1 + a_3 + a_4, -a_3 - a_4)$. Then clearly T is a linear transformation (do not show that).

- Find the standard matrix representation of T.

- Find $Z(T)$ and write it as span

- Find the Range(T) and write it as span

- Does the point $(0, 1, 0) \in$ Range(T)?

QUESTION 4. Let $T : R^3 \to R^3$ be a linear transformation, such that $T(4, 0, 0) = (4, -4, -4), T(1, 1, 1) = (1, -1, -1)$, $T(0, 0, 2) = (2, -2, -2)$

- (a) Find the standard matrix representation of T. (Note $T(e_1)$ is the first column of M, $T(e_2)$ is the second column of M, $T(e_3)$ is the third column of M)

- Use (a) and find $T(2, 3, 5)$

- Use (a) and find all zeros of T (i.e., Z(T)) and write it as span (i.e., Write Ker(T) as span) .

- Find Range(T) and write it as span

- Let $D = \{(a_1, a_2, a_3) \in R^3 \mid T((a_1, a_2, a_3)) = (2, -2, -2)\}$. Describe the elements in D

QUESTION 5. Let P_5 be the set of all polynomials of degree < 5 with coefficients from Z_3. Convince me that $F = \{f(x) \in P_5 \mid f'(1) = 0$ and $f(1) = 0\}$ is a subspace of P_5. Find a basis for F. What is the size of F (i.e., $|F|$).

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